Measuring Inequality
with ordinal data

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Outline

Introduction

- Inequality analysis: origins go back to Pigou and Dalton
  - explicitly tied into welfare: contrast Gini and Lorenz
  - seen as more fundamental than approaches such as Pareto
- But all of this is erected on rather demanding informational structure
  - income, wealth, cardinally measurable and comparable
  - income, earnings usually assumed to be non-negative
- Maybe need a new approach to inequality measurement

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Ingredients of Measurement Problem

- 3 ingredients of the income-inequality measurement problem:
  - the definition of "income"
  - the definition of the "income-receiving unit"
  - method of aggregation
- Same issues arise in cases where "income" is ordinal
- Look at standard income-inequality problem before modelling ordinal-data problem

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Income Inequality

- 3 ingredients:
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  - **"income-receiving unit":** n persons
  - **method of aggregation:** function $X^n \to \mathbb{R}$
- Usually work with $X^n_\mu \subset \mathbb{R}$
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Utility (1)
Cardinalisation and inequality

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  - Already pointed out in Atkinson (1970)
  - Dalton (1920) suggested inequality of (cardinal) utility
  - But if, for all i, you multiply u_i by $\lambda \in (0,1)$ and add $\delta = \mu[1-\lambda]$...
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Is this something different?

- Atkinson and Dalton examples of "aggregation process"
  - How social values are introduced into an inequality-evaluation of income distribution...
  - ...not the inequality-evaluation of a distribution of utilities.
- Sometimes these are equivalent
  - but sometimes not
  - maybe utility has no natural income equivalent?
- Case 1. U depends on x with no agreed monetary valuation
  - quality of life
  - happiness
- Case 2. U depends on x that is categorical:
  - health status
  - level of completed education
  - access to public services

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Categorical variable
Example: Access to Services

	Case 1	Case 2
	n_k	n_k
Both Gas and Electricity	25	0
Electricity only	25	50
Gas only	25	50
Neither	25	0

- Suppose we have no information about needs / usage
- Nevertheless it is clear that Case 1 seems more unequal than Case 2

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Ways Forward?

- We could try to develop dominance criteria based on median
  - Median may be well defined although mean is not
  - what principle should play the role that is played by PoT in income inequality?
- Could try a family of measures using only median
- For such things as happiness could just use arbitrary cardinalisation
  - over large part of domain may be empirically robust
  - psychologists think Likert scales are OK for cardinalising
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Status and Information

- Step 1 is to define status
  - depends on the purpose of inequality analysis
  - depends on structure of information
  - conventional inequality approach only works in narrowly defined information structure
- In some cases a person's status is self-defining
  - income
  - wealth
- In some cases status is defined given additional distribution-free information
  - example: if it is known that utility is $\log(x)$
- In some cases status requires information dependent on distribution
  - GRE
  - TOEFL

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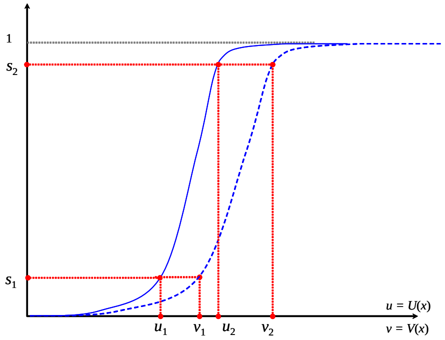
Status and Distribution (1)

Inequality: Ordinal

Cowell, Flachaire

Motivation
Basic Problem
Previous work
Approach
Model
Basic structure
Characterisation
Inequality Measures
Transfer principle
Reference point
Sensitivity
Normalisation
Empirical aspects
Implementation
Performance
Application
Summary

- i's status uniquely defined for a given distribution of u



- disposes of the problem of cardinalisation
  - U and $V = \varphi(U)$ two cardinalisations of the utility of x
  - for each i:u_i and v_i map into s_i

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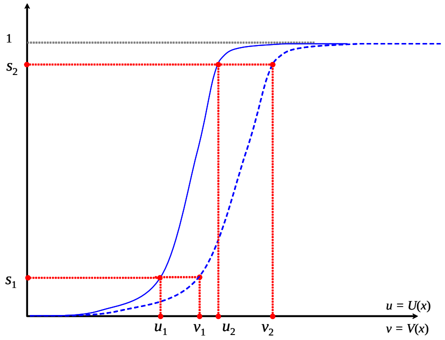

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Status and distribution (2)

- This approach works for categorical data
  - we just have an ordered arrangement of categories $1, 2, ..., k, ..., K$
  - and the numbers in each category $n_1, n_2, ..., n_k, ..., n_K$
- Merger principle
  - merge two adjacent categories that are irrelevant for i
  - then this should leave i's status unaltered
- Merger principle implies that s should be additive in the n_k
  - could have upward-looking ...
  - ... or downward-looking status

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Elements of the Model

- individual's status is given by $s \in S \subseteq \mathbb{R}$
  - status determined from utility using ψ
- vector of status in a population of size n : $\mathbf{s} \in S^n$.
- $e \in S$: an equality-reference point
  - could be specified exogenously
  - could also depend on status vector $e = \eta(\mathbf{s})$
  - η need not be increasing in each component of $\mathbf{s}$
- Inequality: aggregate distance from e
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  - don't need an explicit distance function
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Elements of the Model

- individual's status is given by $s \in S \subseteq \mathbb{R}$
  - status determined from utility using ψ
- vector of status in a population of size n : $\mathbf{s} \in S^n$.
- $e \in S$: an equality-reference point
  - could be specified exogenously
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Basic Axioms

- **[Continuity]** $\succeq$ is continuous on S^n.
- **[Monotonicity in distance]** If $\mathbf{s}, \mathbf{s}' \in S^n_e$ differ only in their ith component then (a) if $s'_i \geq e$:$s_i > s'_i \Longleftrightarrow \mathbf{s} \succ \mathbf{s}'$; (b) if $s'_i \leq e$: $s'_i > s_i \Longleftrightarrow \mathbf{s} \succ \mathbf{s}'$.
- **[Independence]** For $\mathbf{s}, \mathbf{s}' \in S^n_e$, if $\mathbf{s} \sim \mathbf{s}'$ and $s_i = s'_i$ for some i then $\mathbf{s}(\varsigma, i) \sim \mathbf{s}'(\varsigma, i)$ for all $\varsigma \in [s_{i-1}, s_{i+1}] \cap [s'_{i-1}, s'_{i+1}]$.
- **[Anonymity]** For all $\mathbf{s} \in S^n$ and permutation matrix $\mathbf{P}$, $\mathbf{Ps} \sim \mathbf{s}$.

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Standard result

Theorem

Continuity, Monotonicity, Independence, Anonymity jointly imply $\succeq$ is representable by the continuous function $I : S_e^n \to \mathbb{R}$ where $I(\mathbf{s}; e) = \Phi\left(\sum_{i=1}^{n} d(s_i, e), e\right)$, where $d : S \to \mathbb{R}$ is a continuous function that is strictly increasing (decreasing) in its first argument if $s_i > e$ ($s_i < e$).

Corollary

Inequality is total "distance" from equality. Distance d is continuous, satisfies $d(e, e) = 0$. $d(s, e)$ is increasing in status if you move away from the reference point.

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Outline

Structure Theorem

- We need to impose more structure on the problem
- **[Scale irrelevance]** For all $\lambda \in \mathbb{R}_+$: if $\mathbf{s}, \mathbf{s}' \in S_e^n$ and $\lambda\mathbf{s}, \lambda\mathbf{s}' \in S_{\lambda e}^n$ then $\mathbf{s} \sim \mathbf{s}' \Rightarrow \lambda\mathbf{s} \sim \lambda\mathbf{s}'$.

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Impose also Scale irrelevance. Then $\succeq$ is representable by $I(\mathbf{s}; e) = \Phi\left(\sum_{i=1}^n d(s_i, e), e\right)$, where the function d takes the form $(s, e) = e^c \phi\left(\frac{s}{e}\right)$, ϕ is a continuous function and c is an arbitrary constant.

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Characterisation Theorem

- We now impose yet more structure on the problem
- **[Ratio scale irrelevance]** Suppose there are $\mathbf{s} \in S_e^n$ and $\mathbf{s}^\circ \in S_{e^\circ}^n$ such that $\mathbf{s} \sim \mathbf{s}^\circ$. Then for all $\lambda > 0$, $\mathbf{s}' \in S_{e'}^n$ and $\mathbf{s}" \in S_{e"}^n$ such that for each i, $s_i'/e = \lambda s_i/e$ and $s_i"/e = \lambda s_i^\circ/e^\circ$: $\mathbf{s}' \sim \mathbf{s}"$.

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Impose also Ratio scale irrelevance. Then $\succeq$ is representable as $\Phi\left(I\left(\mathbf{s};e\right),e\right)$ where $I_\alpha\left(\mathbf{s};e\right) = \frac{1}{\alpha[\alpha-1]}\left[\frac{1}{n}\sum_{i=1}^n \left[\frac{s_i}{e}\right]^\alpha - c\right]$, $\alpha, c \in \mathbb{R}$ and Φ is increasing in its first argument.

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A Usable Inequality Index?

- Class of functions that could be used as inequality measures:
  - $\Phi(I(\mathbf{s};e), e)$
  - $e = \eta(\mathbf{s})$, the reference point
  - $I(\mathbf{s};e) = I_\alpha(\mathbf{s};\eta(\mathbf{s})) = \frac{1}{\alpha[\alpha-1]}\left[\frac{1}{n}\sum_{i=1}^{n}\left[\frac{s_i}{\eta(\mathbf{s})}\right]^{\alpha} - c(\eta(\mathbf{s}))\right]$
- Key questions:
- Do functions of the form $\Phi(I(\mathbf{s};e), e)$ "look like" inequality measures?
  - transfer principle?
  - reference point?
  - sensitivity to parameters
- What is the appropriate form for Φ ?
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Outline

Transfer Principle (1)

- Standard version of transfer principle is not applicable
  - "Mean status" is not quite like mean income
  - can change in interesting ways
- Can show a property related to transfer principle
  - if e is independent of $\mathbf{s}$
  - or if e depends only on $\mu(\mathbf{s}) = \frac{1}{n}\sum_{i=1}^{n} s_i$
- Then for all α in such cases:
  - if i's status increases $\delta > 0$ and j's status decreases by δ
  - such that $s_i < s_j$ and $s_i + \delta < s_j - \delta$),
  - then inequality is reduced
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Four distributional scenarios (1)

	Case 0		Case 1		Case 2		Case 3	
	n_k	s_i	n_k	s_i	n_k	s_i	n_k	s_i
B	0		25	1	0		25	1
E	50	1	25	$3/4$	50	1	25	$3/4$
G	25	$1/2$	25	$1/2$	50	$1/2$	50	$1/2$
N	25	$1/4$	25	$1/4$	0		0	
$\mu(\mathbf{s})$		$11/16$		$5/8$		$3/4$		$11/16$

- n_k is # persons in category $k \in \{\mathrm{B}, \mathrm{E}, \mathrm{G}, \mathrm{N}\}$
- $s_i = \frac{1}{n}\sum_{\ell=1}^{k(i)} n_\ell$ – *downward*-looking status

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N	25	1	25	1	0		0	
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- n_k is # persons in category $k \in \{\mathrm{B},\mathrm{E},\mathrm{G},\mathrm{N}\}$
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Four distributional scenarios (2)

	Case 0		Case 1		Case 2		Case 3	
	n_k	s_i	n_k	s_i	n_k	s_i	n_k	s_i
B	0		25	1	0		25	1
E	50	1	25	$3/4$	50	1	25	$3/4$
G	25	$1/2$	25	$1/2$	50	$1/2$	50	$1/2$
N	25	$1/4$	25	$1/4$	0		0	
$\mu(\mathbf{s})$		$11/16$		$5/8$		$3/4$		$11/16$

- Case 0 to Case 1:
  - 25 people promoted from E to B
  - if e equals to any of values taken by $\mu(\mathbf{s})$
  - then inequality increases

Four distributional scenarios (2)

	Case 0		Case 1		Case 2		Case 3	
	n_k	s_i	n_k	s_i	n_k	s_i	n_k	s_i
B	0		25	1	0		25	1
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  - if e equals to any of values taken by $\mu(\mathbf{s})$
  - then inequality increases

Four distributional scenarios (3)

	Case 0		Case 1		Case 2		Case 3	
	n_k	s_i	n_k	s_i	n_k	s_i	n_k	s_i
B	0		25	1	0		25	1
E	50	1	25	$3/4$	50	1	25	$3/4$
G	25	$1/2$	25	$1/2$	50	$1/2$	50	$1/2$
N	25	$1/4$	25	$1/4$	0		0	
$\mu(\mathbf{s})$		$11/16$		$5/8$		$3/4$		$11/16$

- Case 0 to Case 2:
  - 25 people promoted from N to G
  - if e equals to any of values taken by $\mu(\mathbf{s})$
  - then inequality decreases

Four distributional scenarios (3)

	Case 0		Case 1		Case 2		Case 3	
	n_k	s_i	n_k	s_i	n_k	s_i	n_k	s_i
B	0		25	1	0		25	1
E	50	1	25	$3/4$	50	1	25	$3/4$
G	25	$1/2$	25	$1/2$	50	$1/2$	50	$1/2$
N	25	$1/4$	25	$1/4$	0		0	
$\mu(\mathbf{s})$		$11/16$		$5/8$		$3/4$		$11/16$

- Case 0 to Case 2:
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  - then inequality decreases

Transfer Principle again

	Case 0		Case 1		Case 2		Case 3	
	n_k	s_i	n_k	s_i	n_k	s_i	n_k	s_i
B	0		25	1	0		25	1
E	50	1	25	$3/4$	50	1	25	$3/4$
G	25	$1/2$	25	$1/2$	50	$1/2$	50	$1/2$
N	25	$1/4$	25	$1/4$	0		0	
$\mu(\mathbf{s})$		$11/16$		$5/8$		$3/4$		$11/16$

- Case 0 to Case 1: inequality increases
- Case 0 to Case 2: inequality decreases
- Case 0 to Case 3: combination results in ambiguous change

Transfer Principle again

	Case 0		Case 1		Case 2		Case 3	
	n_k	s_i	n_k	s_i	n_k	s_i	n_k	s_i
B	0		25	1	0		25	1
E	50	1	25	$3/4$	50	1	25	$3/4$
G	25	$1/2$	25	$1/2$	50	$1/2$	50	$1/2$
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Outline

Reference point

- Inequality index requires a reference point
- **Mean status:** $e = \eta(\mathbf{s}) = \mu(\mathbf{s})$
  - for continuous distributions will equal 0.5
  - for categorical data, there is no counterpart to fixed-mean assumption in income-inequality analysis
- **Median status**: $e = \eta(\mathbf{s}) = \mathrm{med}(\mathbf{s})$
  - not well-defined
  - in the example median is any value in interval $M(\mathbf{s})$
  - $M(\mathbf{s}) = [1/2, 1)$ in cases 0 and 2
  - $M(\mathbf{s}) = [1/2, 3/4)$ in cases 1 and 3
- **Max status**: $e = 1$
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Sensitivity

- α captures the sensitivity of measured inequality
- If α is high $I_\alpha(\mathbf{s};e) = \frac{1}{\alpha[\alpha-1]}\left[\frac{1}{n}\sum_{i=1}^{n}\left[\frac{s_i}{e}\right]^\alpha - c\right]$ sensitive to high status-inequality
- If $\alpha = 0$ and $c = 1$ then becomes $I_0(\mathbf{s};e) = -\frac{1}{n}\log\left(\frac{s_i}{e}\right)$
- If $e = \mu(\mathbf{s})$ and $\alpha = c = 1$ then we have $I_1(\mathbf{s};e) = \frac{1}{n}\sum_{i=1}^{n}\frac{s_i}{e}\log\left(\frac{s_i}{e}\right)$

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Outline

Behaviour of $I_0(\mathbf{s}; e)$

	Case 0	Case 1	Case 2	Case 3
$\mu(\mathbf{s})$	$^{11}/_{16}$	$^{5}/_{8}$	$^{3}/_{4}$	$^{11}/_{16}$
$\text{med}_1(\mathbf{s})$	$^{3}/_{4}$	$^{5}/_{8}$	$^{3}/_{4}$	$^{5}/_{8}$
$\text{med}_2(\mathbf{s})$	$^{1}/_{2}$	$^{1}/_{2}$	$^{1}/_{2}$	$^{1}/_{2}$
$I_0(\mathbf{s};\ \mu(\mathbf{s}))$	0.1451	0.1217	0.0588	0.0438
$I_0(\mathbf{s};\ \text{med}_1(\mathbf{s}))$	0.2321	0.1217	0.0588	-0.0515
$I_0(\mathbf{s};\ \text{med}_2(\mathbf{s}))$	-0.1732	-0.1013	-0.3465	-0.2746
$I_0(\mathbf{s};\ 1)$	0.5198	0.5917	0.3465	0.4184

- $I_0(\mathbf{s};\ \mu(\mathbf{s}))$, $I_0(\mathbf{s};\ \text{med}_1(\mathbf{s}))$: inequality *decreases* when one person promoted from E to B
  - Case 0 to Case1, or Case 2 to Case 3
  - movement changes both the $\mu(\mathbf{s})$ and $\text{med}_1(\mathbf{s})$ ref points
- $I_0(\mathbf{s};\ \text{med}_2(\mathbf{s})) < 0$ for *all* cases in example!
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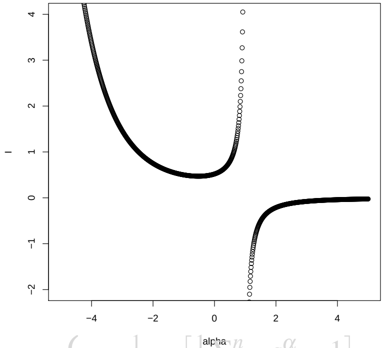
Behaviour of $I_\alpha(\mathbf{s};1)$ with α



- $$I_\alpha(\mathbf{s},1) = \begin{cases} \frac{1}{\alpha(\alpha-1)}\left[\frac{1}{n}\sum_{i=1}^n s_i^\alpha - 1\right], & \text{if } \alpha \neq 0,1, \\ -\frac{1}{n}\sum_{i=1}^n \log s_i. & \text{if } \alpha = 0. \end{cases}$$

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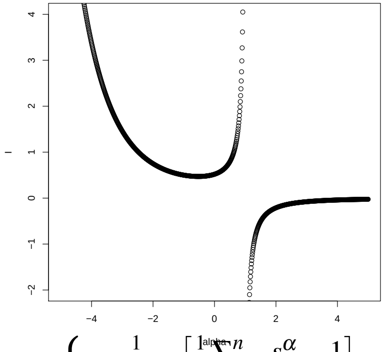

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$I_\alpha(\mathbf{s};1)$: Parameter restriction

- Inequality can also be written $I_\alpha(\mathbf{s},1) = \frac{1}{\alpha-1}\left[\frac{1}{n}\sum_{i=1}^{n}\frac{s_i^\alpha-1}{\alpha}\right]$
- if $0 < s < 1$ then $[s^\alpha - 1]/\alpha < 0$ and if $s = 1$ then $[s^\alpha - 1]/\alpha = 0$
- $I_\alpha(\mathbf{s};1)$ only well behaved under the parameter restriction $\alpha < 1$.
- Alternative representation as Atkinson index on status
- $A_\alpha(\mathbf{s}) := \begin{cases} 1-\left[\frac{1}{n}\sum_{i=1}^{n} s_i^\alpha\right]^{1/\alpha} & \text{if } \alpha<0 \text{ or } 0<\alpha<1, \\ 1-\left[\prod_{i=1}^{n} s_i\right]^{1/n} & \text{if } \alpha=0. \end{cases}$

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Implementation

- Description of sample

$$x_i = \begin{cases} 1 & \text{with sample proportion } p_1 \\ 2 & \text{with sample proportion } p_2 \\ \dots & \\ K & \text{with sample proportion } p_K \end{cases},$$

- Point estimate of the index:
  - $$I_\alpha = \begin{cases} \frac{1}{\alpha(\alpha-1)}\left[\sum_{i=1}^K p_i \left[\sum_{j=1}^i p_j\right]^\alpha - 1\right] & \text{if } \alpha \neq 0,1 \\ -\sum_{i=1}^K p_i \log\left[\sum_{j=1}^i p_j\right] & \text{if } \alpha=0 \end{cases}$$
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Asymptotics

- From the CLT I_α is asymptotically Normally distributed
- Estimator of cov matrix of $(p_1, p_2, \ldots, p_k)$ is

$$\Sigma = \frac{1}{n}\begin{bmatrix} p_1(1-p_1) & -p_1p_2 & \ldots & -p_1p_K \\ -p_2p_1 & p_2(1-p_2) & \ldots & -p_2p_K \\ \vdots & \vdots & \vdots & \vdots \\ -p_Kp_1 & -p_Kp_2 & \ldots & p_K(1-p_K) \end{bmatrix}$$

- $\widehat{\mathrm{Var}}(I_\alpha) = D\Sigma D^\top \qquad \text{with} \qquad D = \left[\frac{\partial I_\alpha}{\partial p_1}; \quad \frac{\partial I_\alpha}{\partial p_2}; \ldots; \quad \frac{\partial I_\alpha}{\partial p_K}\right]$
- $\frac{\partial I_\alpha}{\partial p_l} = \frac{1}{\alpha(\alpha-1)}\left(\left[\sum_{i=1}^{l} p_i\right]^\alpha + \alpha \sum_{i=l}^{K-1} p_i \left[\sum_{j=1}^{i} p_j\right]^{\alpha-1}\right), \alpha \neq 0$
- $\frac{\partial I_0}{\partial p_l} = -\log\left[\sum_{j=1}^{l} p_j\right] - \sum_{i=l}^{K-1} p_i \left[\sum_{j=1}^{i} p_j\right]^{-1}$

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$$\Sigma = \frac{1}{n}\begin{bmatrix} p_1(1-p_1) & -p_1p_2 & \dots & -p_1p_K \\ -p_2p_1 & p_2(1-p_2) & \dots & -p_2p_K \\ \vdots & \vdots & \vdots & \vdots \\ -p_Kp_1 & -p_Kp_2 & \dots & p_K(1-p_K) \end{bmatrix}$$
- $\widehat{\mathrm{Var}}(I_\alpha) = D\Sigma D^\top$ with $D = \left[\frac{\partial I_\alpha}{\partial p_1};\ \frac{\partial I_\alpha}{\partial p_2};\dots;\ \frac{\partial I_\alpha}{\partial p_K}\right]$
- $\frac{\partial I_\alpha}{\partial p_l} = \frac{1}{\alpha(\alpha-1)}\left(\left[\sum_{i=1}^{l} p_i\right]^\alpha + \alpha\sum_{i=l}^{K-1} p_i\left[\sum_{j=1}^{i} p_j\right]^{\alpha-1}\right), \alpha \neq 0$
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Confidence Intervals

- 3 variants of CIs: Asymptotic, Percentile Bootstrap, Studentized Bootstrap
- $CI_{asym} = [I_\alpha - c_{0.975}\widehat{\text{Var}}(I_\alpha)^{1/2}\,;\, I_\alpha + c_{0.975}\widehat{\text{Var}}(I_\alpha)^{1/2}]$
  - $c_{0.975}$ from the Student distribution $T(n-1)$
  - do not always perform well in finite samples
- Bootstraps: generate resamples, $b = 1, \ldots, B$
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- $CI_{perc} = [c_{0.025}^b\,;\, c_{0.975}^b]$
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Performance Test

- Take an example with 3 ordered categories ($K = 3$)
- Samples are drawn from a multinomial distribution with probabilities $\pi = (0.3, 0.5, 0.2)$
- Is asymptotic or bootstrap distribution a good approximation of the exact distribution of the statistic?
  - if we are using 95% CIs of I_α
  - coverage error rate should be close to nominal rate, 0.05
- Check coverage error rate of CIs as sample size increases
  - $\alpha = -1, 0, 0.5, 0.99$
  - 199 bootstraps
  - 10 000 replications to compute error rates
  - $n = 20, 50, 100, 200, 500, 1000$

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	α	-1	0	0.5	0.99
Asymptotic B	$n = 20$	0.0606	0.0417	0.0598	0.0491
	$n = 500$	0.0523	0.0492	0.0521	0.0523
	$n = 1000$	0.0485	0.0540	0.0552	0.0549
Percentile B	$n = 20$	0.0384	0.0981	0.0912	0.1023
	$n = 500$	0.0509	0.0513	0.0552	0.0554
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Studentized B	$n = 20$	0.1275	0.0843	0.1041	0.1377
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World values survey

- Life satisfaction question:

All things considered, how satisfied are you with your life as a whole these days? Using this card on which 1 means you are "completely dissatisfied" and 10 means you are "completely satisfied" where would you put your satisfaction with your life as a whole? (code one number): Completely dissatisfied – 1 2 3 4 5 6 7 8 9 10 – Completely satisfied

- Health question:

All in all, how would you describe your state of health these days? Would you say it is (read out): 1 Very good, 2 Good, 3 Fair, 4 Poor.

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  - Weak relation internationally?
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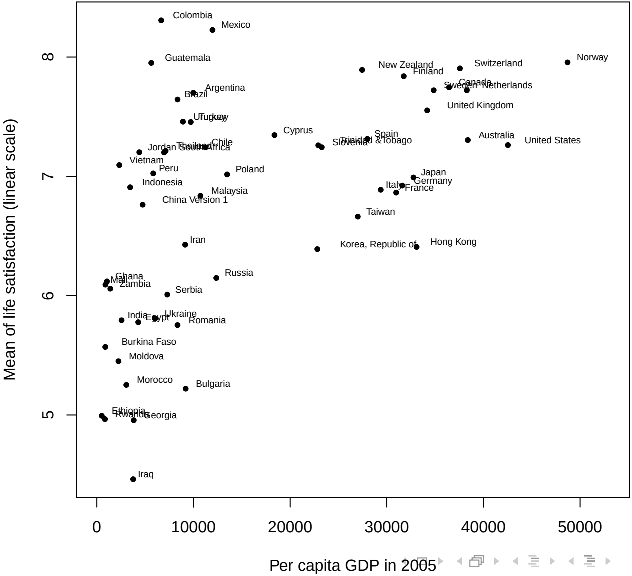
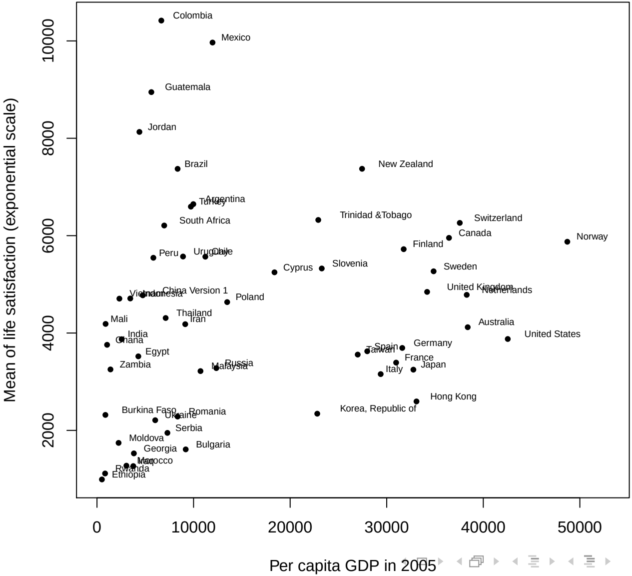
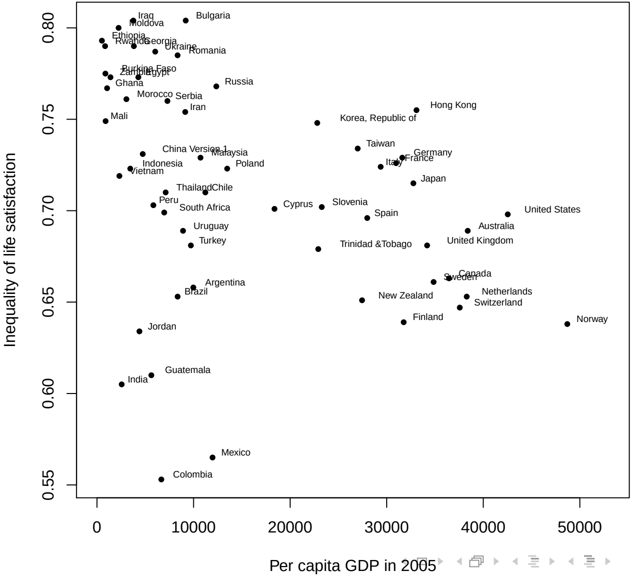
GDP and Life satisfaction (Linear)



GDP and Life satisfaction (Exponential)



GDP and Inequality of Life satisfaction



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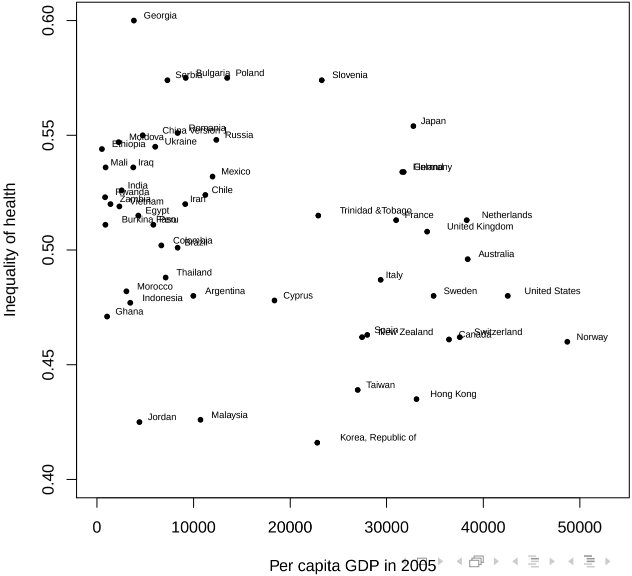

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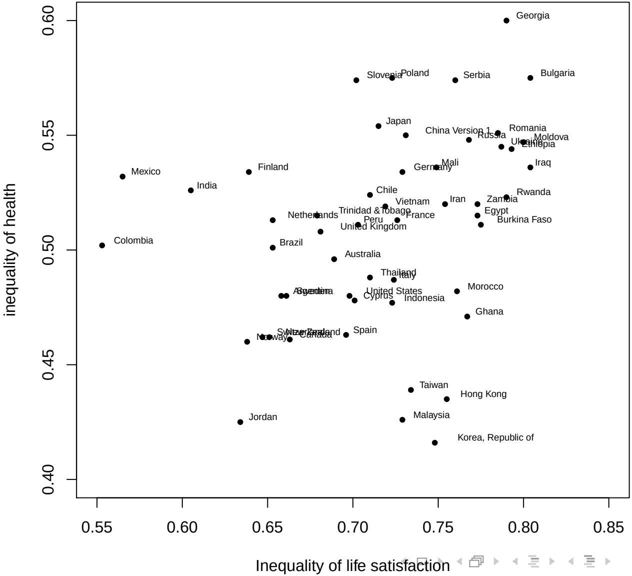

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- Satisfaction / GDP results sensitive to the cardinal interpretation of the answers
  - linear: get a positive relation below \$15 000, flat after that
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- OLS estimate of I_0(life satisfaction) on the GDP per capita small and negative
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- Interpretation in terms of inequality of opportunity
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